mode. This load characteristic was recorded on a high impedance pen-type x-y recorder. The load lines for a given magnetic field are essentially straight, indicating negligible effect of the current upon the internal impedance of the generator. Thus, there is no evidence of increased conductivity due to magnetically induced effects. Actually, no additional nonequilibrium is expected, since the rf electric field is believed to be substantially greater than the induced electric field. An effective Hall parameter of 12.5 at a magnetic field of 0.4 weber/m^2 was computed using $V_{oc} = \beta UBL$, the open circuit voltage from Fig. 2, an effective axial length of 0.5 in. and a flow velocity of 160 m/sec.

Figure 3 shows the theoretical relationship between effective Hall parameter (Hall parameter divided by one plus the ion slip parameter), the electron temperature, and the reduced magnetic field for argon at 500°K. The effect of electron energy distribution was not accounted for in this calculation. The maximum in the Hall parameter is due to ion slip at the higher reduced magnetic fields, whereas the peak with respect to electron temperature is due to the Ramsauer effect. From this it would appear that a highly nonequilibrium, slightly ionized plasma and a large effective Hall parameter may be incompatible. However, this may offer the possibility of Faraday generator operation at high magnetic fields without the segmented electrodes required to minimize Hall currents; the effective conductivity would be reduced by ion-slip effects however. The use of this figure with the corresponding reduced magnetic field of 100 webers-°K-m^{-2}-torr^{-1} and electron temperature 10,000°K indicates relatively good agreement between the theoretical and measured effective Hall parameter.

Similar experiments were conducted in the Faraday mode. The load characteristics are also linear. However, the open circuit voltage is considerably greater than predicted from $U \times B$ considerations alone. No complete explanation is currently available, although this performance is believed to be due to the Hall effect and the generator circular cross section. Potential measurements of each electrode electrically isolated from the rest indicate that the electrodes on one side of the channel essentially are at equal potentials, whereas the opposing electrodes are not. The roles are reversed upon reversing the direction of the magnetic field. This indicates a high Hall effect on one side of the channel and not the other, i.e., two-dimensional effects are important. It should be mentioned that decreasing the pressure to about 0.350 torr results in an open circuit voltage-magnetic field characteristic with two relative maxima; one occurring at about 0.1 weber/m^2 the other occurring at approximately 0.3 weber/m^2. Lowering the pressure further to 0.150 torr gives the same relative maximum at approximately 0.1 weber/m^2 but at approximately 0.3-weber/m^2 oscillations become apparent.

The results of this preliminary study indicate the need for more sophisticated diagnostic techniques to measure the characteristics of rf-generated plasmas. Further effort is required to fully evaluate their potential in MHD devices.

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Integration of Area Coordinates in Matrix Structural Analysis

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IF area coordinates[1] are used in the expansion for the displacements over triangular elements in the analysis of structures by the direct stiffness method, a certain type of integral occurs frequently. The integral is

$$I = \int_A L_1^i L_2^j L_3^k dA \tag{1}$$

where L_1, L_2, L_3 = area coordinates, i, j, k = integers, and A = area of triangle.

The purpose of this brief Note is to present a closed-form solution for the integral. The solution is

$$I = [i!j!k!/(n+2)!]2A \tag{2}$$

where $n = i + j + k$. This integral was obtained by inspection and has been checked for all terms through the sixth-order in n.

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Nonlinear, Rarefied Rayleigh's Problem

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IN order to test the new discrete ordinate method developed by Huang[1] for the nonlinear transient problems, the proposed method is applied to the nonlinear, rarefied Rayleigh problem with heat transfer. It is shown that the proposed method yields results that agree in the small Mach number limit with Cercignani and Sernagiotto's solutions[2]; in the nonlinear case, they agree with that of Chu.[3]

The nonlinear, unsteady Rayleigh flow problem with heat transfer is now considered. A infinite flat plate is on the $y = 0$ plane and is submerged in a monatomic, originally quiescent gas with density n_0 and temperature T_0. The plate is at temperature T_w and is impulsively set in motion in its own plane with constant velocity w. The unsteady problem is one-dimensional in physical space in that the distribution function $f(y;\bar{v};t)$ describing the state of the system depends only on y. The one-dimensional, nonlinear, unsteady B-B-G-K equation for this case is

$$\frac{\partial f}{\partial t} + v_y \frac{\partial f}{\partial y} = \nu(F - f)$$
$$F = n\left(\frac{1}{2\pi RT}\right)^{3/2} \exp\left[-\frac{1}{2RT}(\bar{v} - \bar{u})^2\right] \tag{1}$$

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